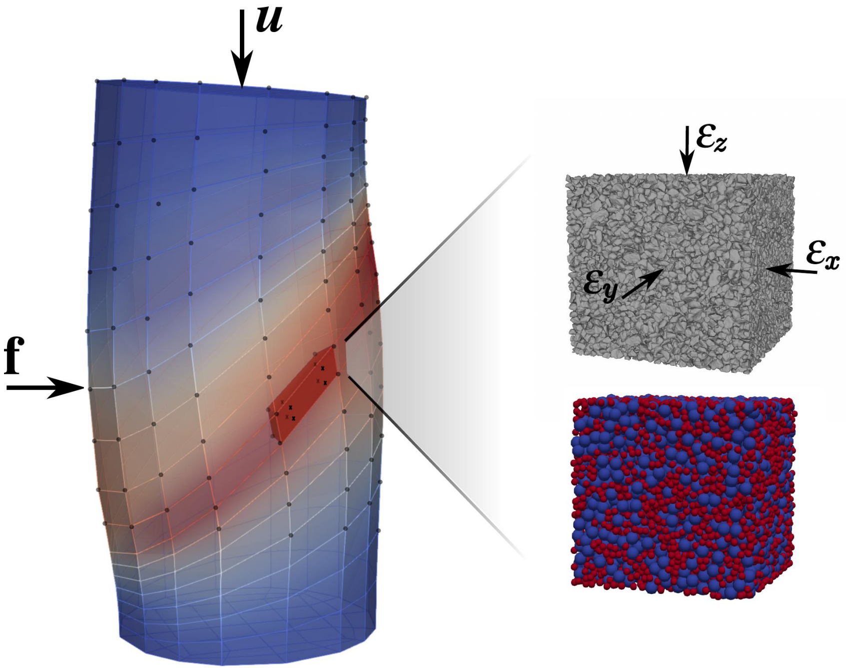


CIVIL-408 Multiscale Modeling in Mechanics

Prof. Kostas Karapiperis

Exercises - Week 5

Project 1 - Random Composite

We will solve this homogenization problem with periodic BCs.

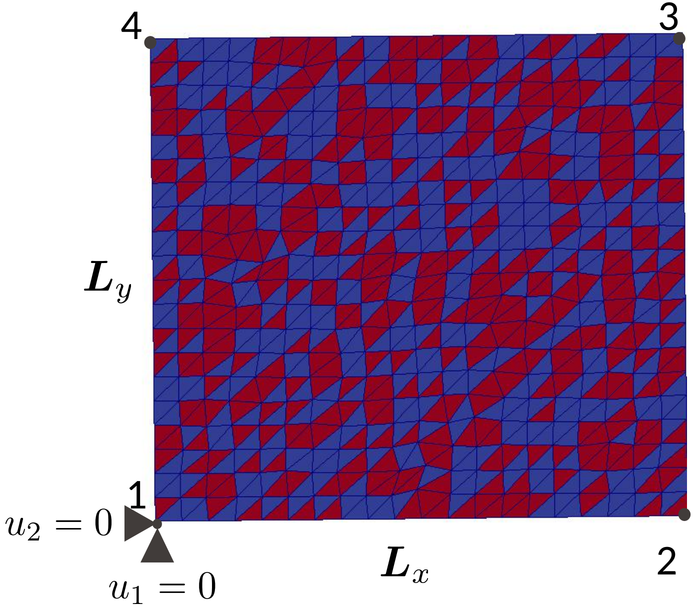
Essential BCs:

$$u^1 = \mathbf{0}$$
$$u^2 = \boldsymbol{\varepsilon} \boldsymbol{L_x}$$
$$u^4 = \boldsymbol{\varepsilon} \boldsymbol{L_y}$$
$$u^3 = \boldsymbol{\varepsilon}\left(\boldsymbol{L_x} + \boldsymbol{L_y}\right)$$

with

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_{11} & 0 \\ 0 & 0 \end{pmatrix}$$

$$\boldsymbol{L}_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mathrm{m} \qquad \boldsymbol{L}_y = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mathrm{m}$$



Project 1 - Random Composite

We will solve this homogenization problem with periodic BCs.

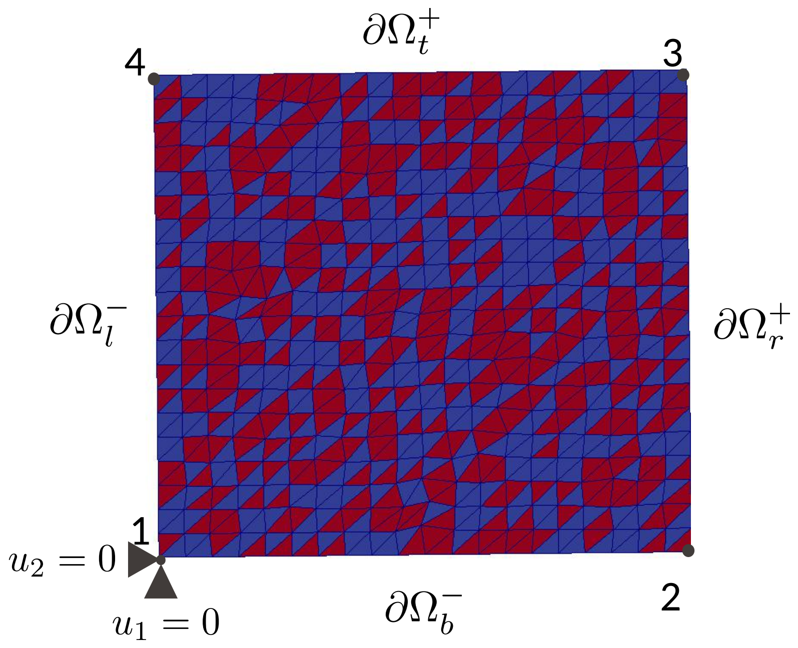
Periodic BCs: $$\boldsymbol{u}^{+}=\boldsymbol{u}^{-}+\boldsymbol{\varepsilon}^{0}\left(\boldsymbol{x}^{+}-\boldsymbol{x}^{-}\right)$$

- right/left: $\boldsymbol{u}^{r}=\boldsymbol{u}^{l}+\boldsymbol{u}^{2}$
- top/bottom: $\boldsymbol{u}^{t}=\boldsymbol{u}^{b}+\boldsymbol{u}^{4}$

This is *already implemented* in the code, but you need to define the **periodic surface pairs**, e.g.

periodicBCs = [$\partial\Omega_r^+$, **1** , **0** , $\partial\Omega_l^-$;
$\partial\Omega_r^+$, **2** , **0** , $\partial\Omega_l^-$]

The + side goes in the first column.



Project 1 - Random Composite

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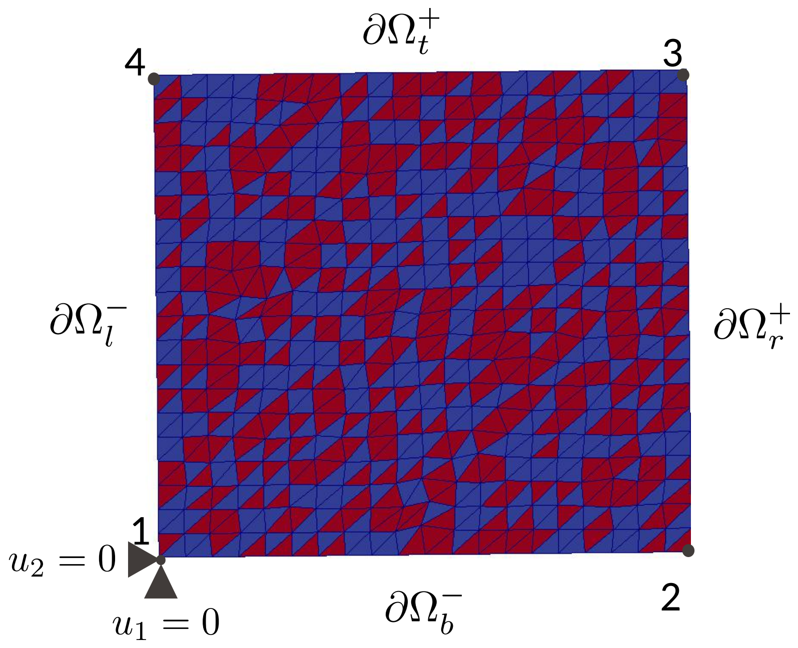


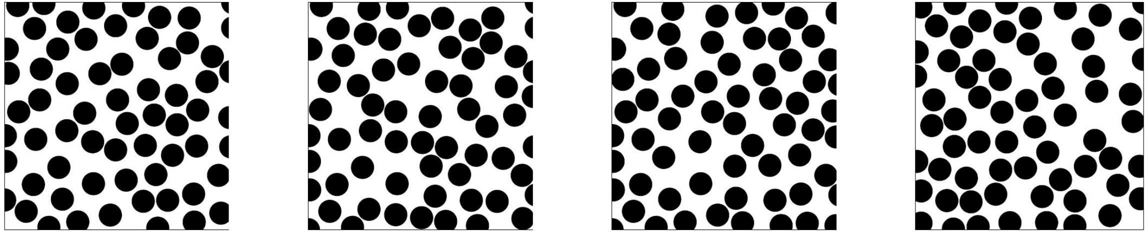
Implementation:

periodicBCs = [42, 0, 0, 22; // node 42 and 22 are periodic in the 0-direction (x)
42, 1, 0, 22] // node 42 and 22 are periodic in the 1-direction (y)

Project 1 - Random Composite

A. Ensemble enlargement

Configurations (or realizations):



Ensemble: a set of multiple realizations

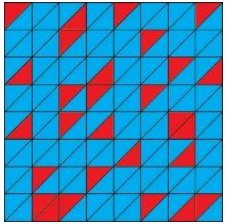
We assume that each configuration produces a response which generally differs between realizations (e.g., some averaged quantity).

Ensemble average of a response over a set of *N* realizations: $\langle\!\langle \cdot \rangle\!\rangle_N = \frac{1}{N}\sum_{i=1}^{N}(\cdot)_i$

Central limit theorem: $\lim_{N\to\infty} \langle\!\langle \cdot \rangle\!\rangle_N = \langle\!\langle \cdot \rangle\!\rangle_\infty$

Project 1 - Random Composite

A. Ensemble enlargement



200x

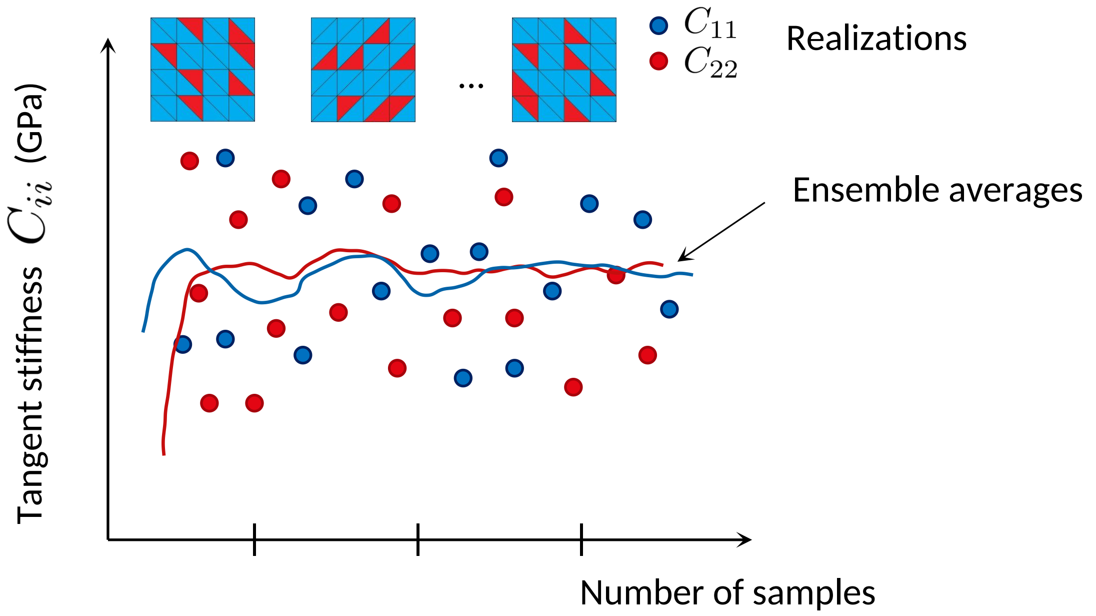
$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} \mathbb{C}^*_{1111} & \mathbb{C}^*_{1122} & \mathbb{C}^*_{1112} \\ \mathbb{C}^*_{2211} & \mathbb{C}^*_{2222} & \mathbb{C}^*_{2212} \\ \mathbb{C}^*_{1211} & \mathbb{C}^*_{1222} & \mathbb{C}^*_{1212} \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{pmatrix}$$

$$\langle\langle \mathbb{C}_{ijkl} \rangle\rangle_\infty$$

Project 1 - Random Composite

K.Karapiperis

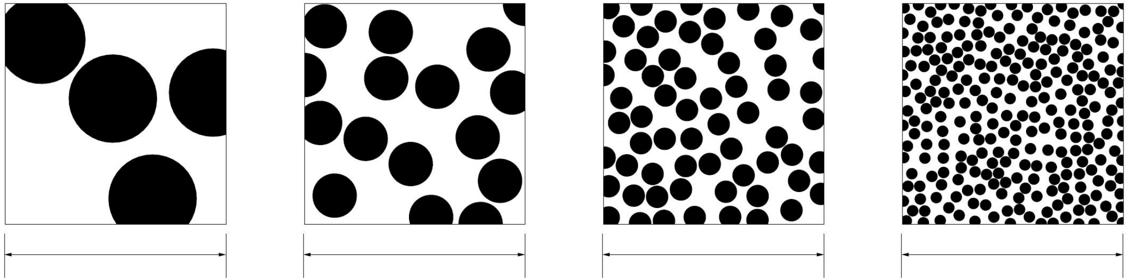
A. Ensemble enlargement



+ Plot ratio to quantify isotropy

B. Sample enlargement

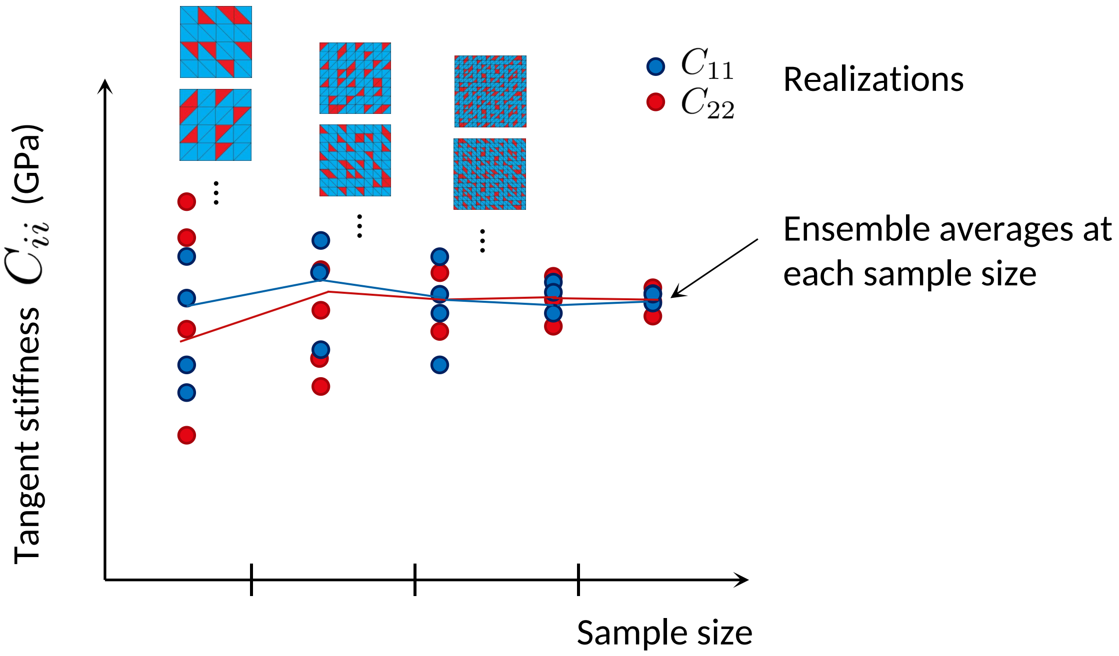
Recall from the lecture:



We continuously increase the size of the RVE until we observe convergence of the moduli.

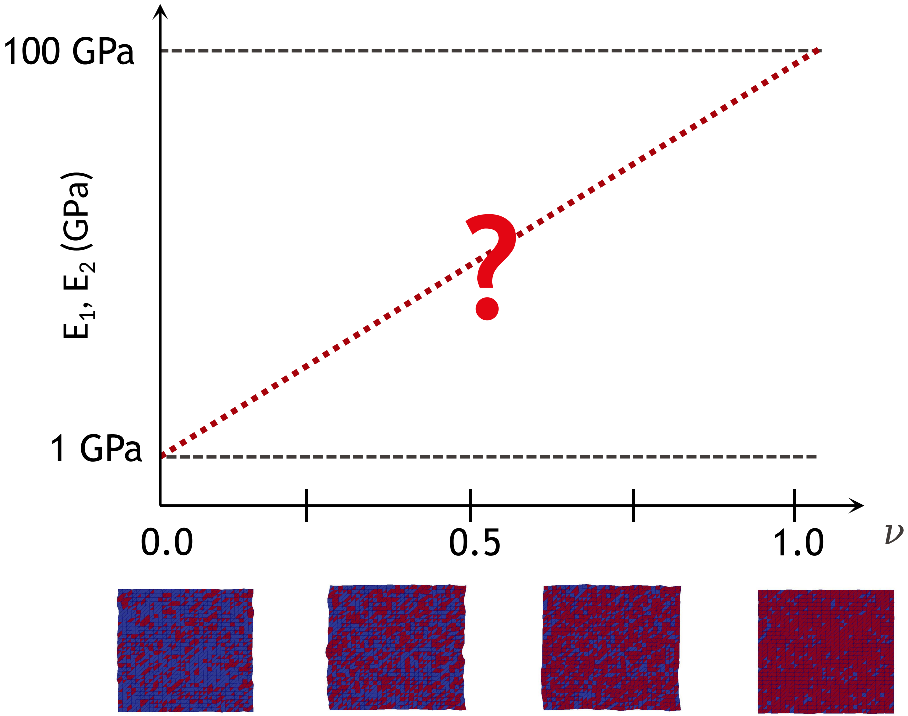
Project 1 - Random Composite

B. Sample enlargement



Project 1 - Random Composite

C. Influence of volume fraction



Project 1 - Random Composite

Hints, problems, sanity checks

1. Poor condition number?

   -> Ensure rigid body motion is prevented - Check essential BCs

2. Does the average strain differ from the imposed one?

3. Stiffness is not (approximately) symmetric?

4. Rotate the RVE by 90°? What do you get?

Project 1 - Random Composite

Let’s move to the Python notebook